No5 略解

$\mathbf{r}_0$ を原点にとって

0 :

$$\mathbf{m} = \frac{1}{2}\int \rho\, d\rho\, d\varphi\, dz\, (\rho\hat{\rho}) \times I\,\delta(\rho - a)\,\delta(z)\,\hat{\varphi}$$

$$= \frac{1}{2} \times 2\pi a^2 I\, \underbrace{(\hat{\rho} \times \hat{\varphi})}_{\hat{z}} = \pi a^2 I\, \hat{z}$$

1、 (a)

$$\mathbf{A} = -\frac{\mu_0}{4\pi} m\hat{z} \times \left(-\frac{1}{r^2}\right)\left(\frac{x}{r}\hat{x} + \frac{y}{r}\hat{y} + \frac{z}{r}\hat{z}\right)$$

$$= \frac{\mu_0 m}{4\pi r^2}\left(\frac{x}{r}\hat{y} - \frac{y}{r}\hat{x}\right)$$

$r = \sqrt{x^2+y^2+z^2}$

$\rho = \sqrt{x^2+y^2}$

$$= \frac{\mu_0 m \rho}{4\pi(\rho^2+z^2)^{3/2}} \underbrace{\left(\frac{x}{\rho}\hat{y} - \frac{y}{\rho}\hat{x}\right)}_{\hat{\varphi}}$$

(b)

$$B_x = \overset{0}{\overset{\shortparallel}{\frac{\partial A_z}{\partial y}}} - \frac{\partial A_y}{\partial z} = \frac{\mu_0 m \rho}{4\pi} \frac{x}{\rho} \frac{3}{\cancel{2}} (\rho^2+z^2)^{-5/2} \cdot \cancel{2}z$$

$$= \frac{3\mu_0 m \rho z}{4\pi(\rho^2+z^2)^{5/2}} \frac{x}{\rho}$$

similarly

$$B_y = \frac{\partial A_x}{\partial z} - \overset{\prime\prime\,0}{\frac{\partial A_z}{\partial x}} = \frac{3\mu_0 m \rho z}{4\pi(\rho^2+z^2)^{5/2}}\,\frac{y}{\rho}$$

and

$$B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = -\frac{\mu_0 m \rho}{4\pi}\,\frac{x}{\rho}\,\frac{3}{2}(\rho^2+z^2)^{-5/2}\,2\rho\,\frac{\partial \rho}{\partial x} \to \frac{x}{\rho}$$

$$-\frac{\mu_0 m \rho}{4\pi}\,\frac{y}{\rho}\,\frac{3}{2}(\rho^2+z^2)^{-5/2}\,2\rho\,\frac{\partial \rho}{\partial y} \to \frac{y}{\rho}$$

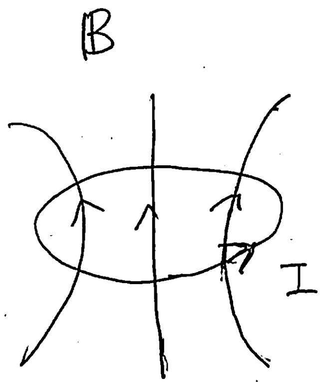
$$+\frac{\mu_0 m}{4\pi}\,\frac{1}{(\rho^2+z^2)^{3/2}}(1+1)$$

$$= \frac{\mu_0 m}{2\pi}\,\frac{1}{(\rho^2+z^2)^{3/2}}\left(1 - \frac{3}{2}\,\frac{\rho^2}{\rho^2+z^2}\right).$$

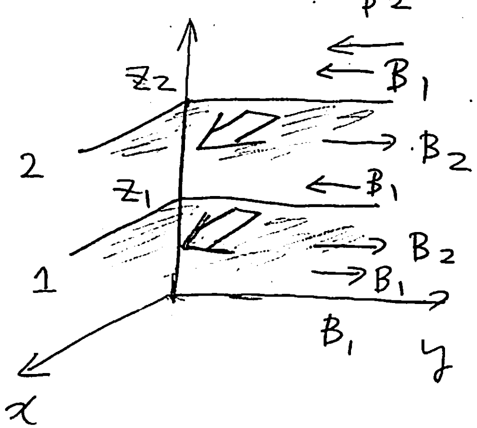
まとめると

$$\mathbb{B} = \frac{3\mu_0 m \rho z}{4\pi(\rho^2+z^2)^{5/2}}\,\hat{\rho} + \frac{\mu_0 m}{2\pi}\,\frac{1}{(\rho^2+z^2)^{3/2}}\left(1 - \frac{3}{2}\,\frac{\rho^2}{\rho^2+z^2}\right)\hat{z}$$

$\left(\hat{\rho} = \frac{x}{\rho}\hat{x} + \frac{y}{\rho}\hat{y}\right.$ を用いた$\left.\right)$.



2.




$K_1>0,\ K_2>0$ として図示。

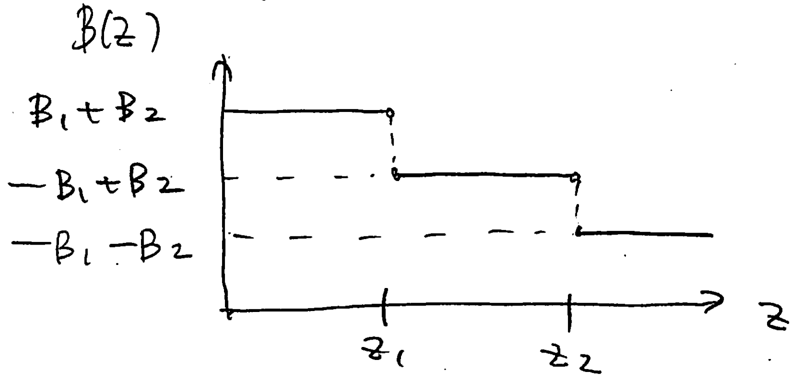
(a) 1, 2が作る磁場はy軸に平行で、強さはそれぞれ

$$B_1=\frac{\mu_0 K_1}{2}\qquad B_2=\frac{\mu_0 K_2}{2}\qquad (\text{No5-3より})$$

向きは図の通り。

(b) $\boldsymbol{B}(\boldsymbol{r})=B(z)\hat{y}$ で

$$B(z)=\begin{cases}B_1+B_2 & (z<z_1)\\ -B_1+B_2 & (z_1<z<z_2)\\ -B_1-B_2 & (z_2<z)\end{cases}$$




(c) 上の結果から

① $z<z_1$ では $T_{yy}=\dfrac{(B_1+B_2)^2}{2\mu_0}$ $T_{xx}=T_{zz}=-\dfrac{(B_1+B_2)^2}{2\mu_0}$ あとの成分は 0

② $z_1<z<z_2$ では $T_{yy}=\dfrac{(-B_1+B_2)^2}{2\mu_0}$ $T_{xx}=T_{zz}=-\dfrac{(-B_1+B_2)^2}{2\mu_0}$ 〃

③ $z_2<z$ では $T_{yy}=\dfrac{(B_1+B_2)^2}{2\mu_0}$ $T_{xx}=T_{zz}=-\dfrac{(B_1+B_2)^2}{2\mu_0}$ 〃

(d) 1の受ける力

面積

$$\vec{F}_1 = K_1 S B_2 \hat{z} = S\frac{\mu_0}{2}K_1K_2$$

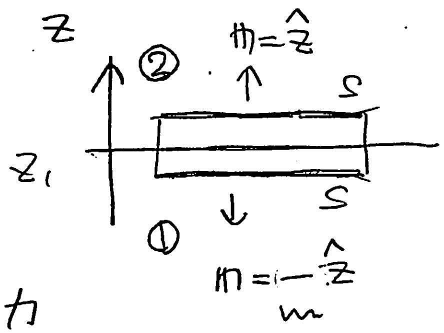
単位面積あたり)

$$\rightarrow \ \vec{F}_1/S = \frac{\mu_0}{2}K_1K_2\hat{z}$$

2の受ける力

$$\vec{F}_2 = K_2 S B_1(-\hat{z}) = -S\frac{\mu_0}{2}K_1K_2\hat{z} \left(= -\vec{F}_1\right)$$

(e)




$$\vec{F}_1/S = \left(T^{②}_{zz} - T^{①}_{zz}\right)\hat{z}$$

$$= \frac{1}{2\mu_0}\left(-(-B_1+B_2)^2 + (B_1+B_2)^2\right)\hat{z} = \frac{\mu_0}{2}K_1K_2\hat{z}$$

Maxwell応力

$$dF_\mu = \sum_\nu n_\nu T_{\nu\mu}\,dS.$$

力の成分　　表面の外向き法線ベクトル

同様にして $\vec{F}_2/S = -\hat{z}\frac{\mu_0}{2}K_1K_2\hat{z}$

で(d)に一致.