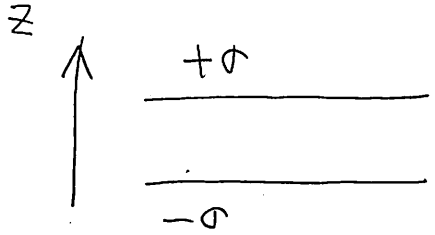
0.

(a) コンデンサーの2つの極板での表面電荷密度は $\pm\sigma$ $(\sigma = Q/S)$



左図のように z 軸をとると*

$$\mathbf{D} = D\hat{z} \qquad D = \sigma$$

電場は

$$\mathbf{E}_1 = E_1\hat{z} \qquad E_1 = D/\epsilon_1 = \sigma/\epsilon_1$$

$$\mathbf{E}_2 = E_2\hat{z} \qquad E_2 = D/\epsilon_2 = \sigma/\epsilon_2$$

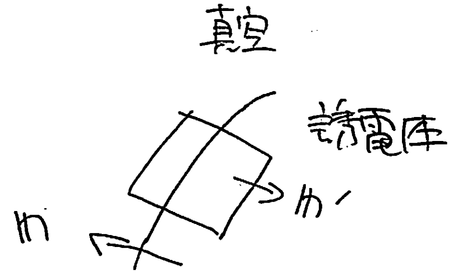
(b) 電位差 V は

$$V = E_1 d_1 + E_2 d_2 = \sigma\left(\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right)$$

$$= \frac{Q}{S}\left(\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right)$$

* 対称性からこの形を仮定し
(端の効果は無視)

$\nabla \cdot \mathbf{D} = \sigma\delta(z)$ を極板付近で体積積分すればわかる



1.

(a) 表面を含む微小領域δVで

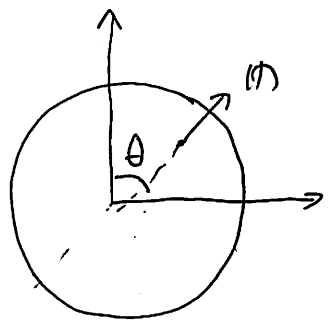
$$\int_{\delta V} dV\, \rho_p(\mathbf{r}) = -\int_{\delta V} dV\, \nabla\cdot\mathbf{P} = -\int_{\delta S} dS\, \mathbf{n}'\cdot\mathbf{P}$$

$$\mathbf{n}' = -\mathbf{n}$$

従って

$$\sigma_p(\mathbf{r}) = \mathbf{n}(\mathbf{r})\cdot\mathbf{P}(\mathbf{r})$$

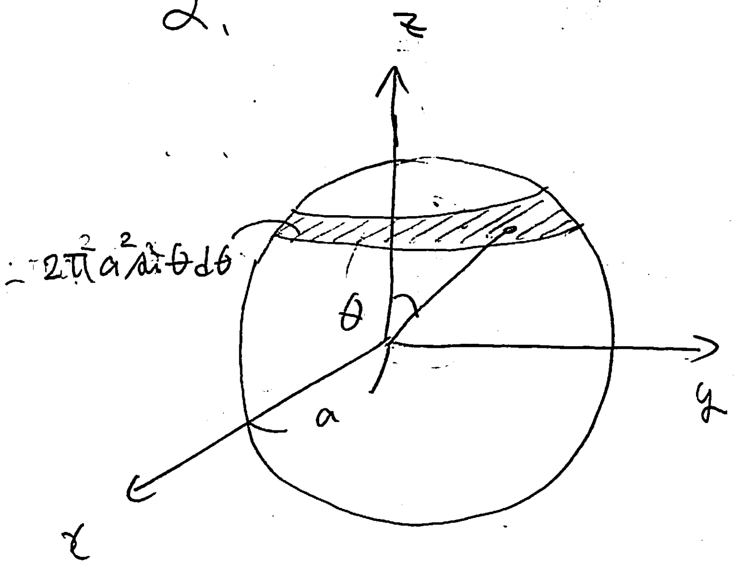
(b)

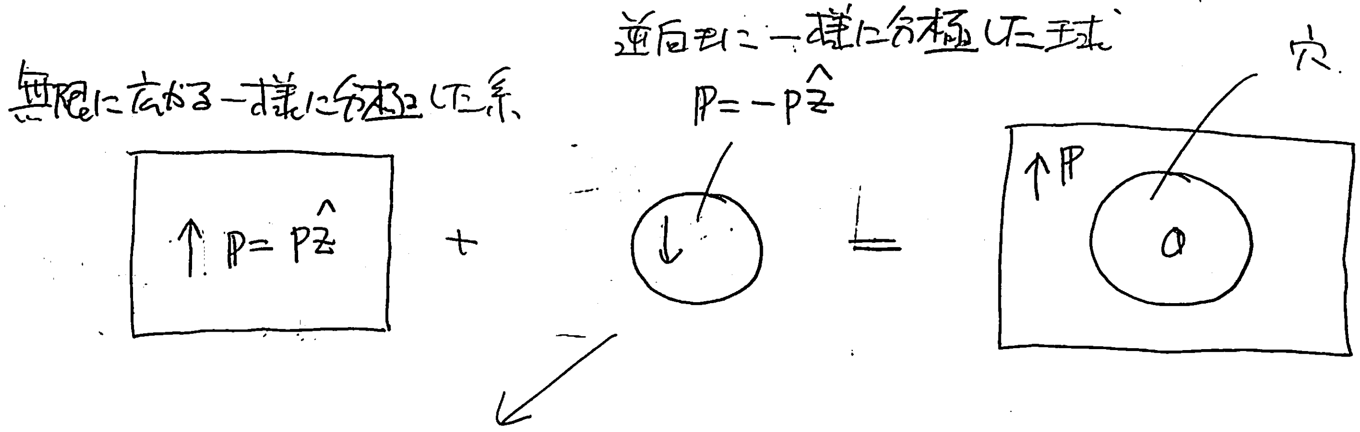


球座標系を用いると

$$\sigma_p = \mathbf{n}\cdot\mathbf{P} = P\cos\theta$$

2.





表面上の点 $(a\cos\theta,\ a\sin\theta\cos\varphi,\ a\sin\theta\sin\varphi)$

その表面分極電荷は $\sigma_p = \mathbf{n}\cdot(-p)\hat{z} = -p\cos\theta$ $(\hat{n} = \hat{r})$

これが穴の中につくる電場は

$$\mathbf{E}(0) = \frac{1}{4\pi\varepsilon_0}\int_0^\pi d\theta\, 2\pi a^2 \sin\theta \frac{-p\cos\theta}{a^2}(-\hat{r})$$

$\hat{r} = \cos\theta\,\hat{z} + \sin\theta\cos\varphi\,\hat{x} + \sin\theta\sin\varphi\,\hat{y}$

$$= \frac{p}{2\varepsilon_0}\int_0^\pi d\theta \cos^2\theta \sin\theta\ \hat{z} = \frac{p}{3\varepsilon_0}\hat{z}$$

3

(1) $\mathbb{E} = E_0 \hat{z}$ $E_0 = \sigma_0/\epsilon_0$ 電位差は $\Delta\phi = E_0 l_z = \dfrac{\sigma_0}{\epsilon_0} l_z = \dfrac{Q}{\epsilon_0}\dfrac{l_z}{l_x l_y}$

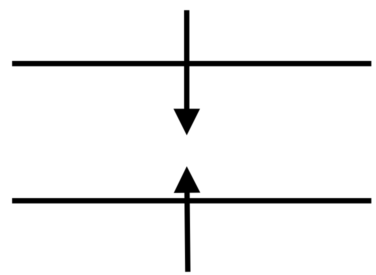
よって 静電容量は. $C = Q/\Delta\phi = \epsilon_0 \dfrac{l_x l_y}{l_z}$

静電場のエネルギーは

$$U_0 = \frac{1}{2}\epsilon_0 E_0^2 l_x l_y l_z =$$

$F_z = -\dfrac{\partial U_0}{\partial l_z} = -\dfrac{1}{2}\epsilon_0 E_0^2 l_x l_y$ 単位面積あたりに直すて $T_0 = \dfrac{F_z}{l_x l_y} = -\dfrac{\epsilon_0}{2}E_0^2$

向きは図の向き.



(2) $D_1 = D_2$ （境界面に垂直なので）従って $E_1 = E_1 \hat{z}$ $\quad E_2 = E_2 \hat{z}$ $\quad E_1 = \sigma/\epsilon_0 = E_0$

$$E_2 = \sigma/\epsilon = E_0 \frac{\epsilon_0}{\epsilon}$$

電位差は $\Delta\phi = E_1(l_z - z) + E_2 z$

$$= \sigma\left[\frac{l_z}{\epsilon_0} + \left(\frac{1}{\epsilon} - \frac{1}{\epsilon_0}\right) z\right] = \sigma \frac{l_z}{\epsilon_0}\left[1 - \left(1 - \frac{\epsilon_0}{\epsilon}\right)\frac{z}{l_z}\right]$$

よって 静電容量は、 $C = Q/\Delta\phi = C_0 \dfrac{1}{1 - \left(1 - \frac{\epsilon_0}{\epsilon}\right)\frac{z}{l_z}}$

静電場のエネルギーは、

$$U = \frac{1}{2}\epsilon_0 E_1^2 l_x l_y (l_z - z) + \frac{1}{2}\epsilon E_2^2 l_x l_y z$$

$$= \underbrace{\frac{1}{2}\epsilon_0 E_0^2 l_x l_y l_z}_{U_0}\left[1 - \frac{z}{l_z} + \underbrace{\frac{\epsilon}{\epsilon_0}\left(\frac{E_2}{E_1}\right)^2}_{\epsilon_0/\epsilon}\frac{z}{l_z}\right]$$

$$= U_0\left[1 - \left(1 - \frac{\epsilon_0}{\epsilon}\right)\frac{z}{l_z}\right]$$

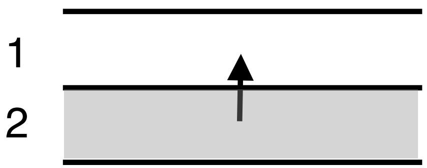
（U_0より減少している）

これから 境界面に働く力は

$$F_z = -\frac{\partial U}{\partial z} = U_0\left(1 - \frac{\epsilon_0}{\epsilon}\right)\frac{1}{l_z}$$

単位面積あたりで

$$T = \frac{F_z}{l_x l_y} = \frac{U_0}{V}\left(1 - \frac{\epsilon_0}{\epsilon}\right)$$



(3) 導体上は等電位になっている → 領域1, 2で電場は同じ $\mathbf{E}_1 = \mathbf{E}_2 = E\hat{z}$

$$\mathbf{E}_1 = \frac{\sigma_1}{\epsilon_0}\hat{z} \qquad \mathbf{E}_2 = \frac{\sigma_2}{\epsilon}\hat{z}$$ なので $$\frac{\sigma_2}{\sigma_1} = \frac{\epsilon}{\epsilon_0} \cdots ①$$

また $$\sigma_1\left(1-\frac{x}{l_x}\right) + \sigma_2\frac{x}{l_x} = \sigma_0 \cdots ②$$ (電荷の保存)

これから

$$\sigma_1 = \frac{\sigma_0}{1+\left(\frac{\epsilon}{\epsilon_0}-1\right)\frac{x}{l_x}}$$

電場は $\mathbf{E}_1 = \mathbf{E}_2 = E\hat{z}$

$$E = E_0\frac{\sigma_1}{\sigma_0} = E_0\frac{1}{1+\left(\frac{\epsilon}{\epsilon_0}-1\right)\frac{x}{l_x}}$$

電位差は

$$\Delta\phi = E\,l_z = \frac{\sigma_1}{\epsilon_0}l_z = \frac{\sigma_0}{\epsilon_0}l_z\frac{1}{1+\left(\frac{\epsilon}{\epsilon_0}-1\right)\frac{x}{l_x}}$$

静電容量は,

$$C = Q/\Delta\phi = C_0\left[1+\left(\frac{\epsilon}{\epsilon_0}-1\right)\frac{x}{l_x}\right]$$

静電エネルギーは

$$U = \frac{1}{2}\epsilon E^2 x\, l_y l_z + \frac{1}{2}\epsilon_0 E^2 (l_x - x) l_y l_z$$

$$= \underbrace{\frac{1}{2}\epsilon_0 E_0^2 l_x l_y l_z}_{U_0}\left(\frac{E}{E_0}\right)^2\left[\frac{\epsilon}{\epsilon_0}\frac{x}{l_x} + \left(1-\frac{x}{l_x}\right)\right]$$

$$E/E_0 = \sigma_1/\sigma_0 = \left[1+\left(\frac{\epsilon}{\epsilon_0}-1\right)\frac{x}{l_x}\right]^{-1}$$

$$= \frac{U_0}{1+\left(\frac{\epsilon}{\epsilon_0}-1\right)\frac{x}{l_x}}$$

境界面に働く力は

$$F_x = -\frac{\partial U}{\partial x} = \frac{U_0}{\left[1+\left(\frac{\epsilon}{\epsilon_0}-1\right)\frac{x}{l_x}\right]^2}\left(\frac{\epsilon}{\epsilon_0}-1\right)\frac{1}{l_x}$$

単位面積あたりの大きさは

$$T = \frac{|F_x|}{l_y l_z} = \frac{U_0}{V_0}\frac{\frac{\epsilon}{\epsilon_0}-1}{\left[1+\left(\frac{\epsilon}{\epsilon_0}-1\right)\frac{x}{l_x}\right]^2}$$

向きは図の向き。
($\epsilon > \epsilon_0$ ならば x を増やす方向)