8

1.

k l .

$$
\begin{vmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
ka_{i1}+lb_{i1} & ka_{i2}+lb_{i2} & \cdots & ka_{in}+lb_{in} \\
a_{n1} & a_{n2} & \cdots & a_{nn}
\end{vmatrix}
$$

$$
= k
\begin{vmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
a_{i1} & a_{i2} & \cdots & a_{in} \\
a_{n1} & a_{n2} & \cdots & a_{nn}
\end{vmatrix}
+ l
\begin{vmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
b_{i1} & b_{i2} & \cdots & b_{in} \\
a_{n1} & a_{n2} & \cdots & a_{nn}
\end{vmatrix}
\tag{201}
$$

- 2

$$
\begin{vmatrix}
1 & 2 \\
2\times1+3\times1 & 2\times2+3\times3
\end{vmatrix}
= 2
\begin{vmatrix}
1 & 2 \\
1 & 2
\end{vmatrix}
+ 3
\begin{vmatrix}
1 & 2 \\
1 & 3
\end{vmatrix}
\tag{202}
$$

3   .

- 2   2

$$
\begin{vmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{vmatrix}
= a_{11}a_{22} - a_{12}a_{21}
$$

$$
= (a_{11}a_{22} - 0\times a_{21}) + (0\times a_{22} - a_{12}a_{21})
$$

$$
=
\begin{vmatrix}
a_{11} & 0 \\
a_{21} & a_{22}
\end{vmatrix}
+
\begin{vmatrix}
0 & a_{12} \\
a_{21} & a_{22}
\end{vmatrix}
\tag{203}
$$

2. 1

n $\hat{A} = (a_{ij})$,

$$
|\hat{A}| = \sum_{\sigma \in P_n} sgn(\sigma) a_{1\sigma(1)} a_{2\sigma(2)} a_{3\sigma(3)} \cdots a_{n\sigma(n)}.
\tag{204}
$$

1   , $\text{sgn}(\sigma)$

.,

$$
|\hat{A}| = \sum_{j=1}^{n} a_{1j}(-1)^{j-1} \sum_{\sigma' \in P_{(n-1)}} \text{sgn}(\sigma') a_{2\sigma(2)} a_{3\sigma(3)} \cdots a_{n\sigma(n)}
$$

$$
= \sum_{j=1}^{n} a_{1j}\tilde{a}_{1j}
\tag{205}
$$

.

$$\tilde{a}_{1j} = (-1)^{j-1} \sum_{\sigma' \in P(n-1)} \text{sgn}(\sigma') a_{2\sigma(2)} a_{3\sigma(3)} \cdots a_{n\sigma(n)}$$

$$= (-1)^{j-1} \begin{vmatrix} a_{21} & a_{22} & \dots & a_{2j-1} & a_{2j+1} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3j-1} & a_{3j+1} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nj-1} & a_{nj+1} & \dots & a_{nn} \end{vmatrix} \quad (206)$$

.

- 2

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$= a_{11}(-1)^{1-1}|a_{22}| + a_{12}(-1)^{2-1}|a_{21}|$$

$$= a_{11}\tilde{a}_{11} + a_{12}\tilde{a}_{12} \quad (207)$$

,

$$\tilde{a}_{11} = (-1)^{1-1}|a_{22}| \quad (208)$$

$$\tilde{a}_{12} = (-1)^{2-1}|a_{21}| \quad (209)$$

- 3

,

$$
\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} 0 & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} 0 & 0 & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}
$$

$$
= (-1)^{1-1} \begin{vmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + (-1)^{2-1} \begin{vmatrix} a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}
$$

$$
+ (-1)^{3-1} \begin{vmatrix} a_{13} & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}
$$

$$
= (-1)^{1-1} \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & a_{23} & a_{33} \end{vmatrix} + (-1)^{2-1} \begin{vmatrix} a_{12} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & a_{23} & a_{33} \end{vmatrix}
$$

$$
+ (-1)^{3-1} \begin{vmatrix} a_{13} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & a_{23} & a_{33} \end{vmatrix}
$$

$$
= a_{11}(-1)^{1-1} \begin{vmatrix} a_{22} & a_{32} \\ a_{23} & a_{33} \end{vmatrix} + a_{12}(-1)^{2-1} \begin{vmatrix} a_{22} & a_{32} \\ a_{23} & a_{33} \end{vmatrix}
$$

$$
+ a_{13}(-1)^{3-1} \begin{vmatrix} a_{22} & a_{32} \\ a_{23} & a_{33} \end{vmatrix}
$$

$$
= a_{11}\tilde{a}_{11} + a_{12}\tilde{a}_{12} + a_{13}\tilde{a}_{13} \tag{210}
$$

3. i

i .   i

, $\mathrm{sgn}(\sigma)$

.,

$$|\hat{A}| = \sum_{j=1}^{n} a_{ij}\tilde{a}_{ij} \tag{211}$$

$$\tilde{a}_{ij} = (-1)^{i+j} \sum_{\sigma' \in P_{(n-1)}} \mathrm{sgn}(\sigma') a_{2\sigma(2)} a_{3\sigma(3)} \cdots a_{n\sigma(n)}$$

$$= (-1)^{i+j} \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1j-1} & a_{1j+1} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j-1} & a_{2j+1} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i-11} & a_{i-12} & \dots & a_{i-1j-1} & a_{i-1j+1} & \dots & a_{i-1n} \\ a_{i+11} & a_{i+12} & \dots & a_{i+1j-1} & a_{i+1j+1} & \dots & a_{i+1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nj-1} & a_{nj+1} & \dots & a_{nn} \end{vmatrix}$$

- 2

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$= a_{22}(-1)^{2+2}|a_{11}| + a_{21}(-1)^{2+1}|a_{12}|$$

$$= a_{22}\tilde{a}_{22} + a_{21}\tilde{a}_{21} \tag{212}$$

$$= a_{11}\tilde{a}_{11} + a_{12}\tilde{a}_{12}$$

$$\tag{213}$$

,

$$\tilde{a}_{21} = (-1)^{2+1}|a_{12}| \tag{214}$$

$$\tilde{a}_{22} = (-1)^{2+2}|a_{22}| \tag{215}$$

- 3

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\tilde{a}_{11} + a_{12}\tilde{a}_{12} + a_{13}\tilde{a}_{13}$$

$$= a_{21}\tilde{a}_{21} + a_{22}\tilde{a}_{22} + a_{23}\tilde{a}_{23}$$

$$= a_{31}\tilde{a}_{31} + a_{32}\tilde{a}_{32} + a_{33}\tilde{a}_{33}$$

4.

n $\hat{A}$ A^{-1}

.

$$\hat{A}^{-1} = \frac{\hat{\tilde{A}}}{|\hat{A}|} \tag{216}$$

$\hat{\tilde{A}}$,

$$\hat{\tilde{A}} = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{21} & \dots & \tilde{a}_{n1} \\ \tilde{a}_{12} & \tilde{a}_{22} & \dots & \tilde{a}_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{1n} & \tilde{a}_{2n} & \dots & \tilde{a}_{nn} \end{pmatrix} \tag{217}$$

.

• 2

$$\hat{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \tag{218}$$

,   .

$$
\begin{aligned}
|\hat{A}|I_2 &= \begin{pmatrix} |\hat{A}| & 0 \\ 0 & |\hat{A}| \end{pmatrix} \\
&= \begin{pmatrix} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{11} & a_{12} \end{vmatrix} \\ \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{21} & a_{22} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix} \\
&= \begin{pmatrix} a_{11}\tilde{a}_{11} + a_{12}\tilde{a}_{12} & a_{11}\tilde{a}_{21} + a_{12}\tilde{a}_{22} \\ a_{21}\tilde{a}_{11} + a_{22}\tilde{a}_{12} & a_{21}\tilde{a}_{21} + a_{22}\tilde{a}_{22} \end{pmatrix} \\
&= \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{21} \\ \tilde{a}_{12} & \tilde{a}_{22} \end{pmatrix} \\
&= \hat{A}\hat{\tilde{A}} \tag{219}
\end{aligned}
$$

$|\hat{A}|$ $\hat{A}^{-1}$

$$\hat{A} = \frac{\hat{\tilde{A}}}{|\hat{A}|} \tag{220}$$